Critical Reflections on the Hafele and Keating Experiment

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In 1971 Hafele and Keating performed their famous experiment which confirmed the time dilation predicted by SRT by use of macroscopic clocks.
As it had already been shown [1], the continuation of reasoning applied by Hafele and Keating leads to the absurd conclusion that the Earth is not rotating around the Sun. Hafele and Keating derived a proper formula starting from false reasoning and this is the origin of the paradox. They tried to derive the formula from SRT, while the proper derivation can only be obtained from GRT [2]. There were also serious doubts concerning the experimental part of their work [4,5], but it does not matter now because today the GPS system confirms what H&K wanted to prove. Finally, H&K wanted to confirm SRT but their experiment confirmed, in fact, the GRT.
If we take a closer look at other experiments confirming SRT, we will come to the conclusion that all the experiments in fact confirm GRT similarly to the H&K experiment, because in order to compare times in two reference frames we have to disturb motion of one of the frames and this brings the problem to problems described by GRT. The pure inertial motion makes observation from two observed each other frames fully symmetrical, and we are not able to define in which of the frames the time flies slower. In this case we can draw the conclusion that the slowing of time in pure inertial motion can be only an observational effect. Only the change of speed of one of the participants transforms the observed time dilation into the real one. Therefore we can ask the questions – are there any serious experimental evidences confirming SRT? And - if in order to register the shortening of time the change of speed is necessary – can we assume, as it is done in SRT, that time slows down as a function of velocity.? Maybe the slowing of the time is rather a function of the change in velocity than the velocity itself?

In my previous papers [1,2], I was writing about the paradox resulting from the experiment preformed by Hafele and Keating.
The idea of this experiment was to compare indications of atomic clocks positioned in three coordinates systems moving in relation to one another: in the jets traveling around the world to the west and east, and on the Earth's surface. Hafele and Keating wanted to prove the predictions of SRT with the help of macroscopic clocks.

Solution of any problem with the help of SRT is possible only in the frame of inertial observer. Since all the clocks taking part in the experiment are performing a rotational motion around the Earth's axis, Hafele and Keating decided to accept a risky assumption that the inertial frame should be positioned on the Earth's pole. The assumption was risky because according to SRT the coordinates system should be bound with one of the clocks taking part in the experiment [3]. The binding of the inertial system with the Earth's pole was connected with adding the velocity of rotational motion of the Earth around it's axis to the velocity of each of the considered clocks. This should change the predicted results of the experiment and thus, according SRT, such assumption should not be allowed. However, Hafele and Keating argued that what is not allowed for linear motion can be allowed for the rotational one.

They started from the SRT formula describing the time dilation as a function of velocity:

(1) $$\tau_0 = t\left(1 - \frac{\omega^2 r^2}{2c^2}\right)$$

where

$$\sqrt{1 - \frac{(\omega r)^2}{c^2}} \cong 1 - \frac{(\omega r)^2}{2c^2}$$

τ_0 – time of the clock on the Earth's surface
τ – time indicated by the moving clock
t – time of the hypothetical clock on the Earth's pole
ω - Earth's angular velocity
r – radius of the Earth

Finally, the change of time resulting from the difference of height of the clocks - gh/c^2 - was added, and the formula took a following form:

(2) $$\tau - \tau_0 = \tau_0\left[-\left(\pm 2\omega r v + v^2\right)\Big/ 2c^2 + gh\Big/c^2\right]$$

where

v - velocity of plane in relation to the Earth's surface
h- height of the flight in relation to the Earth's surface

The formula, although based on the risky assumption, is correct (see appendix for detailed derivation). However, binding the inertial coordinates system with the center of rotation allows to accept, as the inertial frame, also another center of rotation; for instance, the pole of the Sun. Such an assumption will change the velocities of all of the clocks taking part in the experiment, by a great component (approx. 100 times higher than the velocities of the clocks in relation to the Earth's axis), and this should change the predicted indications of the clocks. In fact, such change of the rotation's center brings the change of predicted results by several thousand percent, giving a very strong dependence of the predicted results on the hour of the start and landing – see formula (3).

(3) $$\tau - \tau_0 \cong \{(\pm 2\omega r v + v^2) + 2(\omega r \pm v)V\cos[(\omega \pm v/r)(t - t_0)] - 2\omega r V\cos[\omega(t - t_0)] + 2gh\}\tau_0/2c^2$$

where V – velocity of the Earth rotation around the Sun
t_0 – time of the plane's takeoff
t – time (h:min:sec) of the day

The absence of such dependence, during the experiment, together with an assumption that the reasoning of Hafele and Keating is correct, leads to the absurd conclusion that V=0 (in eq. 3), which is equivalent to the statement that the Earth is not rotating around the Sun. Moreover, binding, in turn, the reference frame with the center of the galaxy, we are coming to the analogous conclusion that the galaxy is not rotating either etc., etc.

The correct solution of the problem can only be obtained if we are considering the problem according to the GRT.
We are starting from the description of motion in the constant gravitational field.

(4) $$\tau = \sqrt{g_{00}}\sqrt{1-\frac{V^2}{c^2}}t$$

where

$$g_{00} = 1-\frac{2MG}{rc^2}$$

M – mass of the Earth
r – radius of the Earth
τ – time indicated by the moving clock
t – the time at an infinite distance from the Earth – in the reference frame in which the gravitational field of the Earth is constant.
Substituting for V in formula (4) respectively the velocities of clocks in jets and the clock on the Earth's surface in relation to the hypothetical reference frame on the Earth's pole, we are also obtaining the formula (2).
The formula derived on the basis of GRT is identical with the one derived by Hafele and Keating. However, following the assumption of the constant gravitational field, the reference frame connected with the Sun or with the centre of the galaxy must rotate together with the Earth (rotating around the Sun or the center of the galaxy) and it prevents the addition of any velocity, resulting from the rotational motion, to the velocities of the clocks. Hence, the problem of the clocks is not the problem of the inertial motion any more, but the problem of the non-inertial motion which can be properly solved only with the help of GRT. Therefore, the Hafele and Keating experiment did not confirm the predictions of SRT, which was the authors' intention. Apart from the doubts concerning their experimental method and the method of interpretation of the experimental results [2,4,5], their experiment confirmed only the predictions of GRT. On the other hand, the fact of the time dilation during the flight around the Earth is confirmed every day by the GPS system. However, the formula describing this time dilation is derived on the basis of GRT in a way similar to the one shown in the Appendix.

The paper [1] proving, on the basis of the Hafele and Keating experiment, that the Earth is not rotating around the Sun, could be treated as a joke or a ridicule. However, the problem that Hafele and Keating encountered, seems to be more serious that it appears.

The analysis of the Hafele and Keating experiment and its conclusions encouraged me to take a closer look at other experiments confirming predictions of SRT. It proved that, similarly to the Hafele and Keating experiment, other experiments have proved, in fact, GRT while experimental confirming of SRT seems to remain impossible.

I would like to start the consideration from a discussion of a completely symmetrical problem: two twins in two identical rockets are in empty space. In a certain moment they are taking off in opposite directions and are moving with identical velocities. After reaching a certain distance, a randomly selected twin is turning back and after some time he catches up with the second twin. Then they are comparing their clocks. Let's notice that in the described case both twins were mutually observing the time dilation in their frames, but finally the change of the velocity was the main phenomena, deciding which of them was younger. As

long as the twins were moving with a uniform motion, they were mutually observing the time dilation in their frames but it was not possible to check if their times were really flowing slower or whether the slowing of time was only a seeming result being the effect of the observation.
The condition which is **necessary** to register the real time dilation in the observed frame is the change of the line, along which the events are simultaneous, being the result of the change of the velocity [6]. Hence the randomly chosen twin, whose rocket has turned back, will register a shorter time than the twin still moving with the uniform motion.
During the analysis of the twin paradox, or other similar phenomena, only the time dilation resulting from the SRT is taken into account, while the moment in which the velocity had been changed is ignored.

Meanwhile, as was mentioned before, if the **observed** time dilation is to become the **real** dilation, it is **necessary** to take into consideration the moment in which the change of velocity took place. It seems that this moment can not be ignored during the analysis of the phenomenon.
The above remark is not related to the quantitative description, because the results obtained with the help of SRT are in accordance with the measurements. We are talking now about the qualitative description which, when the point in which the velocity is changed is ignored, gives us the false justification of the change of the time. Finally, such a false justification hinders further attempts at understanding such enigmatic physical variable as time and, as it has been shown on the example of the Hafele and Keating experiment, can lead to various paradoxes.

Now let us come back to the Hafele and Keating experiment described above, in which the attempt at describing non-inertial motion with the help of SRT led to a series paradoxical conclusions.
If we are describing a non-inertial motion (in case of the Hafele and Keating experiment it is a rotational motion around the Earth's axis) with a help of differential equations, then in fact we are describing series of infinitesimally small segments, along which the body is moving with uniform motion according to SRT, and a series of points, between this segments, in which the lines, denoting events that are simultaneous, are changing.
Hafele and Keating in their theory, treating the problem only with the SRT apparatus, took into account only the motion along these segments, while the points between the segments in which the lines, denoting events that are simultaneous, are changing, had been neglected – fig. 1. Finally, they obtained a correct solution – similarly as it has been obtained in the case of the twins paradox described above – but the result they obtained was based on a false reasoning and this allowed for drawing a series of absurd conclusions.

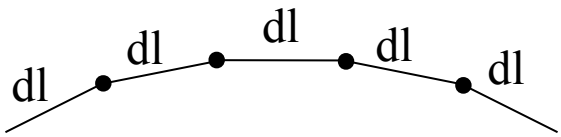

Fig.1 No- inertial motion in the differential notation consists of a series of infinitesimally small segments on which the inertial motion takes place.

Let's notice that ignoring, in the description of the twins paradox, the point at which the line denoting events that are simultaneous is changing, we are doing quite the same mistake as

Hafele and Keating did in their experiment. In the case of the twins paradox, drawing any such spectacular absurd conclusion, as in the case of the Hafele and Keating experiment, has not succeeded yet. However, the mechanism of description of the twins paradox and description of the clocks' motion in the Hafele and Keating experiment are identical – the mistakes are made, but due to their agreement with the experimental data, it is difficult to discover them.
Finally, we are still assuming that the change of the time flow in the moving frame is only the result of the velocity, while the true physical reasons for this phenomena can be quite different.
The relationship between the velocity and the change of time flow is unquestionable; however, there exists additionally, a still ignored effect of changing of the lines, denoting events that are simultaneous, which is **necessary** to change the **observed** time change into the **real** time change and which decides which participant of the experiment will age at a slower rate.
Just the analysis of the influence of the moment in which the line, denoting events that are simultaneous, is changing, on the flow of the time, can be a clue for understanding the nature of such an enigmatic physical variable as time. Hence, analogously as in the case of the Hafele and Keating experiment, the problem of the twins paradox and all other similar problems should be described with the help of GRT, taking into account the point in which the change of the line denoting events that are simultaneous takes place, and not only with the help of SRT.

Conclusions.
As it has been shown in this paper, the Hafele and Keating experiment proved the GRT theory, while it was intended to prove the SRT. All the other experiments intended to confirm SRT, in reality probably also confirmed GRT instead of SRT.
The experimental comparison of times indicated by clocks, lifetimes of particles etc., requires, in practice, the disturbing of motion of one of the frames taking part in the experiment. Hence, regardless of the fact that while performing the experiment we can get an impression that we are proving SRT, we are in fact proving GRT. It is not possible to compare clocks without such a disturbance, therefore we will never be able to analyze the "pure" SRT problem.
The only proof for "pure" SRT is the constancy of the speed of light in all coordinates systems. However, recently, in the model of Euclidean Reality alternative to Relativity Theory [7,8], the constancy of the speed of light has been justified without the necessity of assuming the deformation of the space-time.
Therefore, it seems to be reasonable to take a more detailed look at the problems of the moving bodies and analysis of the influence of the moment when the velocity is changing on the fact of the slowing of time in moving frames.

Appendix – Derivation of the H&K formula on the basis of SRT and GRT

Incorrect derivation on the basis of SRT (Hafele and Keating)
Hafele and Keating assumed that the time indicated by the clock positioned on the Earth's surface should be described by the formula:

(5) $$\tau_0 = t\left(1 - \frac{\omega^2 r^2}{2c^2}\right)$$

where

$$\sqrt{1 - \frac{(\omega r)^2}{c^2}} \cong 1 - \frac{(\omega r)^2}{2c^2}$$

τ_0 – time of the clock on the Earth's surface
τ – time indicated by the moving clock
t – time of the hypothetical clock on the Earth's pole
ω - Earth's angular velocity
r – radius of the Earth

Next, the time indicated by the clock in the plane should be described by the formula:

(6) $$\tau = t\left(1 - \frac{(\omega r \pm v)^2}{2c^2}\right)$$

where:

v – the velocity of the plane in relation to the Earth's surface.

When we divide formula (5) by (6) we get:

$$\tau_0\left(1 - \frac{(\omega r \pm v)^2}{2c^2}\right) = \tau\left(1 - \frac{\omega^2 r^2}{2c^2}\right)$$

and finally:

(7) $$\tau - \tau_0 = -\left(\pm 2\omega rv + v^2\right)\frac{\tau_0}{2c^2}$$

for the obtained time change, the change of time resulting from the difference of height of the clocks - gh/c^2 - was added
where:
h – height of the flight of the plane
g – the Earth's acceleration

The final formula has the form:

(8) $$\tau - \tau_0 = \tau_0\left[-\left(\pm 2\omega rv + v^2\right)\Big/2c^2 + gh\Big/c^2\right]$$

Correct derivation on base of GRT

If one would like to obtain the formula in the correct way, it would be necessary to start from the GRT, from the formula describing the time of the body moving in constant gravitational field:

$$\tau = \sqrt{g_{00}}\sqrt{1-\frac{V^2}{c^2}}t \tag{9}$$

where

$$g_{00} = 1-\frac{2MG}{rc^2}$$

M – mass of the Earth

r – radius of the Earth

τ – time indicated by the moving clock

t – the time at an infinite distance from the Earth – in the reference frame in which the gravitational field of the Earth is constant.

V – velocity of a clock in relation to the center of the Earth which is equal to ωr for the clock on the surface of the Earth and $\omega r \pm v$ for the clock in the jet.

Here the time of the clock on the surface of the Earth is described with the formula:

$$\tau_0 \cong t\left(1-\frac{MG}{rc^2}\right)\left(1-\frac{\omega^2 r^2}{2c^2}\right) \tag{10}$$

while the time indicated by the clock in the jet is described by the formula:

$$\tau \cong t\left(1-\frac{MG}{(r+h)c^2}\right)\left(1-\frac{(\omega r \pm v)^2}{2c^2}\right) \tag{11}$$

hence, dividing (11) by (10) we obtain for week gravitational field, low velocities and $h<<r$:

$$\frac{\tau}{\tau_0} = \frac{\left(1-\frac{MG}{(r+h)c^2}\right)\left(1-\frac{(\omega r \pm v)^2}{2c^2}\right)}{\left(1-\frac{MG}{rc^2}\right)\left(1-\frac{\omega^2 r^2}{2c^2}\right)} \cong 1-\frac{MG}{(r+h)c^2}-\frac{(\omega r \pm v)^2}{2c^2}+\frac{MG}{rc^2}+\frac{\omega^2 r^2}{2c^2} \cong \tag{12}$$

$$\cong 1+\frac{MGh}{r^2c^2}-\frac{1}{2c^2}\left(\pm 2\omega r v + v^2\right)$$

and from the above we instantly obtain the formula (8)

$$\tau - \tau_0 = \tau_0\left[-\left(\pm 2\omega r v + v^2\right)\Big/2c^2 + gh\Big/c^2\right] \tag{13}$$

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